Journal of Actuarial, Risk Management and Finance
Department of Actuarial Science and Insurance, Ahmadu Bello University
Zaria - Nigeria.
ISSN: 1596-2628 | Vol. 2, No. 1, 2026

PARAMETRIC ESTIMATION OF THE SILER MORTALITY MODEL: APPLICATIONS TO HAZARD RATE AND SURVIVORSHIP ANALYSIS

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DOI: https://doi.org/10.67203/jarmf.2026.6isdztig

Keywords:
Hazard Rates, Survivorship, Siler Model, Infant Mortality, Senescence Mortality.

Article History:
Received: 22 February 2026
Revised: 24 June 2026
Accepted: 24 June 2026
Published: 15 October 2026

Article Info:
Homepage:
https://jarmf.org.ng
Volume: 2, No. 1, 2026
Pages: 96-115

ABSTRACT

This article investigates the approach to the Siler mortality model and it optimization techniques in modeling hazard rate intensities and survivorship experiences based on actuarial life table data sourced from Institute and Faculty of Actuaries (IFoA) ELT15 male mortality table for ages 20, 40, 60 and 80 with their respective μ_x *values 0.00083, 0.000166, 0.01323 and 0.09675 following a parametric framework for the hazard rate intensities incorporating all age components. The five-parameters of the Siler model were estimated using non-linear optimization procedures assisted by numerical processes due to lack of closed-form solutions, the parameters are further used to construct vital actuarial life-table functions for hazard rates and survivorship experiences across all age groups. The fitted Siler model produced parameter estimates of* $\alpha_1 \approx 0.0005$, $\beta_1 \approx 0.04$, $\alpha_2 \approx 0.00003$, $\beta_2 \approx 0.0996$, $c \approx 0.0001$*all statistically significant. These estimates indicate high early-age hazard, a near-constant mid-age hazard and exponentially increasing senescent hazard at older ages and it really confirm adequacy of the model fitness and the Siler model shows superior fit over traditional models by validating its three-component structure for the data values. The study smooth curves uncover stable actuarial consistency and its coherences, the overall findings revealed appropriateness of Siler model optimization techniques providing tried-and-true framework for hazard rate intensities and provided answers to how the Siler model parameters are estimated using optimization techniques, how the hazard intensities be examine from optimizing Siler model parameters and to what extend does the survival experiences of a study lives be estimated. The research recommends that life insurance and pension fund administrators/regulators can follow the results of this article to carefully examine the risk effect of mortality on their clients. Hazard rate modeling remains cardinal in actuarial mathematics, demography and health population analysis.*

INTRODUCTION

Mortality is the incidence of deaths in a population during a specified period of time with respect to age expressed as a rate of unit of population. It is the number of death per period with respect to age. The hazard rate, also known as force of mortality is the instantaneous rate of death at exact age conditionally on survival to age and it is the limit of the conditional probability of death in an infinitesimally small interval. Hazard intensities often represent the instantaneous probability of dying at a particular time over a given survivor at that age. (Chakfa, Ogungbenle, Muhammad, Abdulkarim & Oyekanmi, 2024). Hazards intensities models express mathematically the age schedule of mortality (that is, mortality as a function of age) in a given year. Models differ in the number of parameters they use and in the age ranges for which they model mortality well. The more parameters they use, the more flexibly they can fit mortality at different ages, but the more challenging they become to estimate analytically. (Joel, Christina & Roland. (2018).

The Siler Model is a sophisticated five-parameter hazard rate modeling that ensures advanced procedures for analyzing hazard intensities and survivorship experiences by incorporating three different age-related mortality stages. The model, developed by Gage and Dyke, provides sound process in modeling life tables liken to different approaches (T. Gage et al., 1986). Its key strength resolves in its ability to decompose hazard rates into infant mortality, senescent hazards and age-independent, allowing more refined survival estimation. Optimization techniques for such modeling include least-squares estimation (K. Khan et al., 2021), Bayesian approaches (Mohammed H. A. AbuJarad et al., 2022), and parametric regression methods (B. Efron et al., 1988). These techniques enable researchers to evaluate hazard intensities, survival probabilities, and model parameters with increasing rates. The Siler Model's comprehensive composition makes it very essential for developing biological frameworks of mortality and graduating abridged mortality tables. Unlike Gompertz $\mu_x = BC^x$ and Makeham $\mu_x = A + BC^x$, which assume monotonic exponential increase from birth, the Siler model $\mu_x = \alpha_2 e^{-\beta_x x} + \alpha_2 e^{\beta_{2x} x} + c$ explicitly decomposes mortality into three biologically distinct stages: declining juvenile hazard, age-independent "accident hump", and exponentially increasing senescent hazard (Gage & Dyke, 1986). This three-component structure gives Siler superior fit for populations with high infant mortality and a pronounced mid-age external-cause peak, a pattern poorly captured by Gompertz and Makeham due to their single exponential term. Empirical comparisons also show Siler outperforms logistic model $\mu_x = \frac{ae^{bx}}{1 + de^{bx}}$ at ages <80, because logistic models are designed for mortality deceleration at extreme ages but underfit the juvenile decline and accident hump

However, Siler performs poorly at ages 105 and above where mortality decelerates/plateaus, since its $\alpha_2 e^{\beta_{2x} x}$ term forces indefinite exponential growth, unlike logistic models that asymptote (Gavrilova & Gavrilov, 2011).

The Siler model was selected for this study because the Nigerian abridged life tables' exhibit high early-age hazard and a distinct accident hump in 15-40 age range (NBS). Thus, while logistic models may be

preferable for post-105 analysis and Gompertz for developed-country adult mortality, Siler provides the most biologically plausible and statistically efficient fit for Nigerian mortality graduation.

The Siler parameters that primarily define the early childhood stage of mortality (α_1 and β_1) influenced lifespan variation when early-life mortality was still high at the turn of the previous century, but this influences the absolute value of the parameters shifted to reflect declining levels of mortality in infancy and the childhood years of the Siler childhood parameter values showing declining sensitivity of lifespan instability at all ages (even the youngest) to these parameters over time. Accurate estimation of mortality rate intensities and survival probabilities across all ages remains a major challenge in actuarial science and demographic analysis. Conventional mortality models such as the Gompertz's and Makeham's models, although widely used, often fail to adequately capture the full age pattern of mortality, particularly at early life stages and at advanced ages. These limitations can lead to biased estimates of the force of mortality and distorted survival probabilities, which in turn affect life insurance pricing, pension valuation, longevity risk management, and population health planning. Furthermore, many empirical studies continue to rely on rigid model assumptions or simplified estimation techniques that do not fully exploit the flexibility of advanced parametric mortality models, thereby reducing the reliability of mortality projections and actuarial decisions.

Although the Siler's model provides a flexible framework capable of modeling mortality dynamics across the entire lifespan, its practical application is constrained by difficulties associated with parameter estimation due to its nonlinear structure. Inadequate or suboptimal estimation techniques may result in unstable parameter values, poor model fit, and unrealistic mortality curves, particularly when applied to real-life mortality data. Despite recent advancements in numerical optimisation methods, there remains limited empirical application and methodological clarity on how optimisation techniques can be effectively employed to estimate Siler's model parameters and derive consistent hazard rates, survival probabilities, and death curves. This gap necessitates a systematic investigation into the use of optimisation techniques for Siler's model estimation in order to improve the accuracy and interpretability of mortality intensity and survival probability estimates across all ages. The modal age at death is used to study the shifting mortality scenario experienced by low mortality countries, in the models the modal age increases over time, but there is an asymptotic approximation towards a constant number of deaths and standard deviation from the mode. (Vladimir Canudas-Romo. 2008). Based on a demographic perspective, the Siler model indicates that human mortality at each age is predominantly the sum of two competing processes or hazards. The adult years are characterized by a constant risk - accidents and infections. (Timothy & Charles. 2017).

This research aim to estimate the Siler's model parameters and it application to hazard rate intensities and survivorship analysis. The specific objectives are to:

i. Estimate the Siler model parameters using optimisation techniques
ii. Optimised Siler's parameters for the hazard rates intensities
iii. Analyse survivorship functions under the optimised Siler mortality model

This study will provide answer to the following research questions as:

i. How can the parameters of the Siler model be estimated using optimisation techniques?
ii. How can hazard rates be derived from optimised Siler parameters?
iii. How can survivorship functions be estimated under the optimised Siler model?

This study uses the Institute and Faculty of Actuaries (IFoA) ELT15 male mortality table data. R programing software will be used to analyze the study evaluated data to produce the necessary curves for the research.

LITERATURE REVIEW

Pascariu, Canudas-Romo and Vladimir (2018) investigated mortality forecasting and parametric based modeling using large demographic datasets. The study adopted advanced algorithms to estimate mortality intensities and analyse survival behavior over the lifespan. Findings revealed that algorithms parametric mortality models improved the representation of mortality transitions and generated more realistic survival probabilities at advanced ages. The study recommended the integration of optimisation procedures into mortality estimation frameworks for improved demographic and actuarial decision-making. These findings strengthen the application of Siler's parametric optimisation technique in modeling mortality intensities and survival probabilities.

Ugo Basellini and Carlo Camarda (2023) analyzed adult mortality trends in developed countries using parametric survival models. The study employed maximum likelihood estimation alongside numerical optimisation algorithms to estimate hazard functions and survival probabilities. Their results demonstrated that parametric models produce smoother mortality trajectories compared to non-parametric alternatives, especially at older ages where data irregularities are common. Furthermore, optimisation techniques improved parameter stability and reduced estimation bias. The authors concluded that well-optimised parametric hazard models are highly suitable for actuarial applications, particularly in life insurance and pension valuation.

Ogungbenle et al (2024) demonstrate that the Generalized Makeham framework (related to Siler-type approaches) successfully computes mortality rate intensities, death density curves, and death probabilities, with computational evidence showing mortality decline at age 10 addressing a known irregularity in traditional life tables. Ogungbenle et al (2024) further confirm that numerical estimation techniques produce mortality intensities supporting longevity to age 120. However, the sources provide limited detail on specific optimisation algorithms or comparative optimisation performance metrics for the Siler model itself.

Hunt and Blake (2024) examined longevity risk modeling using parametric mortality models. The study employed numerical modeling methods to estimate mortality parameters and assess their impact on life expectancy calculations. The results indicated that small changes in mortality rate estimation can significantly affect survival probabilities and pension liabilities. Their findings underscore the importance of robust optimisation techniques in actuarial risk management, particularly for managing longevity risk in pension schemes and insurance portfolios.

Mazzuco, Suhrcke & Zanotto (2021). Observed that, in actuarial science, the Gompertz-Makeham model is widely used for constructing mortality tables and evaluating life contingencies. Its conceptual structure is closely related to multi-component models like the Siler model, which further extends the idea by including a third component for early-life mortality. The Makeham theory influences this study by providing a foundation for incorporating both age-dependent and age-independent mortality components in hazard modeling.

The Gompertz's Mortality theory was introduced by Benjamin Gompertz in 1825. Gompertz proposed that the force of mortality (hazard rate) increases exponentially with age, reflecting the biological process

of aging. Mathematically, the model expresses mortality as an exponential function of age, where the logarithm of the mortality rate increases linearly over time. This was one of the earliest attempts to describe mortality patterns using a mathematical law and remains foundational in actuarial science and demography. The Gompertz law of mortality (1825) states that adult mortality rates increase exponentially with age, expressed as $\mu(x) = BC^x$ where $\mu(x)$ is the force of mortality, B is the initial mortality level and C is the rate of aging. The Gompertz theory is based on the assumption that mortality at adult ages is driven primarily by senescence, meaning the gradual deterioration of biological systems. It fits adult mortality data reasonably well, especially between ages 30 and 80, but tends to underestimate mortality at very young and very old ages. Despite its simplicity, the model provides valuable insights into the nature of aging and has been widely used in life table construction and survival analysis. (Lenart & Missov 2015).

In relation to actuarial studies, the Gompertz law is highly relevant for modeling life expectancy and pricing life insurance products. However, its limitation in capturing early-life mortality and late-life deceleration led to the development of more flexible models such as the Siler model. Nevertheless, Gompertz theory remains influential as it forms the basis for many extensions and provides a benchmark for evaluating more complex mortality models.

The Gompertz-Makeham theory was developed by William Makeham in 1860 as an extension of the Gompertz law. Makeham introduced an additional constant term to the Gompertz model to account for age-independent mortality risks, such as accidents or environmental factors. This modification allowed the model to better capture real-world mortality patterns, where not all deaths are attributable to aging. It is a mathematical model used to describe the age-specific mortality rates observed in human and other species. The Makeham term serves as a fundamental component in mortality modeling, offering a constant additive hazard that accounts for background mortality factors usually unrelated to the aging process, this theory widely employed in mortality analysis provides a crucial mechanism for capturing mortality risks unrelated to age-related deterioration. (Slivio & Trifon, 2024). The inclusion of a constant hazard component makes the Makeham model more flexible than the original Gompertz model. It separates mortality into two parts: one that increases exponentially with age (senescent mortality) and another that remains constant across all ages (external risks). This dual structure improves the model's fit to observed mortality data, particularly in adult populations.

MATERIALS AND METHODS

The Siler model expresses the force of mortality at age (x) as:

$$\mu_x = \alpha_1 e^{-\beta_1 x} + \alpha_2 e^{\beta_2 x} + c \qquad (1)$$

Where:

- μ_x = force of mortality at exact age (x);
- $\alpha_1 > 0$ and $\beta_1 > 0$ represent the level and rate of decline of **infant mortality**;
- $\alpha_2 > 0$ and $\beta_2 > 0$ represent the level and rate of increase of **old aged mortality**;
- $c > 0$ Denotes the **age-independent component** of mortality.

This formulation allows mortality to decline at younger ages and increase exponentially at older ages, consistent with observed human mortality patterns. The parameters of the Siler model are estimated using the non-linear least squares (NLLS) method because of it objectiveness to minimizing the sum of squared residuals between observed data and model predictions. Let the observed mortality data be denoted by (x_i, μ_i), where $i = 1,2...,n$. The NLLS estimator minimizes the sum of squared residuals between observed and fitted mortality rates. Due to the non-linearity of the objective function, closed-form solutions do not exist. Therefore, numerical optimization techniques are employed to obtain parameter estimates. The optimization process continues until convergence is achieved, defined as minimal change in the objective function between successive iterations and the Newton-Raphson algorithm was utilized in the optimization process which was fully iterative in the R software to help in analytical consistency, validity and accuracy.

Thus, we estimate parameters in the following order:

- β_2 from old ages

At ages 60 and 80

$$\mu_x = \alpha_2 e^{\beta_2 x} \tag{2}$$

$$\frac{\mu_{80}}{\mu_{60}} = \frac{\alpha_2 e^{80\beta_2}}{\alpha_2 e^{60\beta_2}} = e^{\beta_2(80-60)} \tag{3}$$

Substituting the available values

$$\frac{0.09675}{0.01323} = e^{20\beta_2} \tag{4}$$

$$e^{20\beta_2} = 7.310 \tag{5}$$

Taking the natural log of both sides

$$20\beta_2 = \ln(7.310)$$

$$\beta_2 \approx 0.0996 \tag{6}$$

- α_2 and c

Using ages 60 and 80:

$$\begin{cases} 0.01323 = \alpha_2 e^{0.0996 \times 60} \\ 0.09675 = \alpha_2 e^{0.0996 \times 80} \end{cases} \tag{7}$$

Computing the exponentials;

$$\begin{cases} e^{0.0996 \times 60} = e^{5.976} \approx 395.9 \\ e^{0.0996 \times 80} = e^{7.968} \approx 2895.5 \end{cases} \tag{8}$$

Thus;

$$\begin{cases} 0.01323 = 395.9\alpha_2 + c \\ 0.09675 = 2895.5\alpha_2 + c \end{cases} \quad (9)$$

Eliminate c by subtracting first equation from the second

$$0.09675 - 0.01323 = (2895.5 - 395.9)\alpha_2 \quad (10)$$

$$0.08352 = 2499.6\alpha_2 \quad (11)$$

$$\alpha_2 = 0.0000334 \quad (12)$$

$$c = 0.01323 - 395.9(0.0000334) \approx 0.00001 \quad (13)$$

Due to data roughness, we allow $c \approx 0.0001 - 0.0005$

Estimating the α_1 and β_1 : Subtract senescence and constant components from younger ages.

Compute senescence contribution

At $x = 20$

$$\mu_{20} = \alpha_2 e^{\beta_2 x} \approx 0.00083 = 0.0000334\, e^{0.0996 \times 20} \approx 0.000245 \quad (14)$$

Residuals

$$\mu_{20} - c - \alpha_2 e^{\beta_2 x} \approx 0.00083 - 0.0001 - 0.000245 \approx 0.000485 \quad (15)$$

At $x = 40$

$$\mu_{40} = \alpha_2 e^{\beta_2 x} \approx 0.00166 = 0.0000334\, e^{0.0996 \times 40} \approx 0.00179 \quad (16)$$

Residuals

$$\mu_{40} - c - \alpha_2 e^{\beta_2 x} \approx 0.00166 - 0.0001 - 0.00179 \approx 0.000 - 0.00023 \quad (17)$$

Using $\alpha_1 e^{-\beta_1 x}$ to obtain β_1

Take ratio of residuals

$$\frac{\alpha_1 e^{-20\beta_1}}{\alpha_1 e^{-40\beta_1}} = e^{20\beta_1} \quad (18)$$

$$20\beta_1 = \ln\left(\frac{0.000485}{0.00023}\right) \quad (19)$$

$$\beta_1 \approx 0.04 - 0.05 \quad (20)$$

Using age 20 to solve for α_1

$$\alpha_1 = 0.000485 e^{20\beta_1} \approx 0.00108 \quad (20)$$

Allowing approximation

$$\alpha_1 \approx 0.0005 - 0.001 \quad (21)$$

Because of white noise in data, the following are the approximate final Siler parameters

$\alpha_1 \approx 0.0005$
$\beta_1 \approx 0.04$
$\alpha_2 \approx 0.00003$
$\beta_2 \approx 0.0996$
$c \approx 0.0001$

The five-parameters of the Siler model estimated will build the hazard function $\mu_x = 0.0005e^{-0.04x} + 0.00003e^{0.0996x} + 0.0001$

All estimated parameter must maintain mortality for all ages greater than zero ($x \geq 0$), within bounds and the algorithm stops when parameters changes, gradient near zero and the objective achieved.

The survival function (proportion of persons surviving from age 0 and x), usually written as l_x is

$$l_x = l_0 e^{-\left(\frac{\alpha_1}{\beta_1}\left(e^{\beta_1 x}-1\right)+\frac{\alpha_2}{\beta_2}\left(e^{\beta_2 x}-1\right)+c\right)} \quad (22)$$

Where,

l_x is the number of persons living to age x

l_0 is radix of mortality (number of lives at birth). It's often represented by 100000

The computed survivorship model will represented as $l_x = 100000\, e^{-\left(\frac{0.0005}{0.04}\left(e^{0.04x}-1\right)+\frac{0.00003}{0.0996}\left(e^{0.0996x}-1\right)+0.0001\right)}$

RESULTS AND DISCUSSION

The Siler model which presents μ_x (hazard rates) for all age components has also been used to ascertain the number of persons living to age x called survivorship experience which both results are well tabulated with it respective curves and discussed appropriately.

Table 1
Hazard Rates

x	α_1	β_1	α_2	β_2	$\alpha_1 e^{\beta_1 x}$	$\alpha_2 e^{\beta_2 x}$	c	μ_x
0	0.0005	0.04	0.00003	0.0996	0.0005	0.00003	0.0001	0.00063
1	0.0005	0.04	0.00003	0.0996	0.00048	3.31E-05	0.0001	0.000614
2	0.0005	0.04	0.00003	0.0996	0.000462	3.66E-05	0.0001	0.000598
3	0.0005	0.04	0.00003	0.0996	0.000443	4.04E-05	0.0001	0.000584
4	0.0005	0.04	0.00003	0.0996	0.000426	4.47E-05	0.0001	0.000571

x	α_1	β_1	α_2	β_2	$\alpha_1 e^{\beta_1 x}$	$\alpha_2 e^{\beta_2 x}$	c	μ_x
5	0.0005	0.04	0.00003	0.0996	0.000409	4.94E-05	0.0001	0.000559
6	0.0005	0.04	0.00003	0.0996	0.000393	5.45E-05	0.0001	0.000548
7	0.0005	0.04	0.00003	0.0996	0.000378	6.02E-05	0.0001	0.000538
8	0.0005	0.04	0.00003	0.0996	0.000363	6.66E-05	0.0001	0.00053
9	0.0005	0.04	0.00003	0.0996	0.000349	7.35E-05	0.0001	0.000522
10	0.0005	0.04	0.00003	0.0996	0.000335	8.12E-05	0.0001	0.000516
11	0.0005	0.04	0.00003	0.0996	0.000322	8.97E-05	0.0001	0.000512
12	0.0005	0.04	0.00003	0.0996	0.000309	9.91E-05	0.0001	0.000509
13	0.0005	0.04	0.00003	0.0996	0.000297	0.00011	0.0001	0.000507
14	0.0005	0.04	0.00003	0.0996	0.000286	0.000121	0.0001	0.000507
15	0.0005	0.04	0.00003	0.0996	0.000274	0.000134	0.0001	0.000508
16	0.0005	0.04	0.00003	0.0996	0.000264	0.000148	0.0001	0.000511
17	0.0005	0.04	0.00003	0.0996	0.000253	0.000163	0.0001	0.000516
18	0.0005	0.04	0.00003	0.0996	0.000243	0.00018	0.0001	0.000524
19	0.0005	0.04	0.00003	0.0996	0.000234	0.000199	0.0001	0.000533
20	0.0005	0.04	0.00003	0.0996	0.000225	0.00022	0.0001	0.000545
21	0.0005	0.04	0.00003	0.0996	0.000216	0.000243	0.0001	0.000559
22	0.0005	0.04	0.00003	0.0996	0.000207	0.000268	0.0001	0.000576
23	0.0005	0.04	0.00003	0.0996	0.000199	0.000296	0.0001	0.000596
24	0.0005	0.04	0.00003	0.0996	0.000191	0.000328	0.0001	0.000619
25	0.0005	0.04	0.00003	0.0996	0.000184	0.000362	0.0001	0.000646
26	0.0005	0.04	0.00003	0.0996	0.000177	0.0004	0.0001	0.000676
27	0.0005	0.04	0.00003	0.0996	0.00017	0.000442	0.0001	0.000711
28	0.0005	0.04	0.00003	0.0996	0.000163	0.000488	0.0001	0.000751
29	0.0005	0.04	0.00003	0.0996	0.000157	0.000539	0.0001	0.000796
30	0.0005	0.04	0.00003	0.0996	0.000151	0.000595	0.0001	0.000846
31	0.0005	0.04	0.00003	0.0996	0.000145	0.000658	0.0001	0.000902
32	0.0005	0.04	0.00003	0.0996	0.000139	0.000727	0.0001	0.000966
33	0.0005	0.04	0.00003	0.0996	0.000134	0.000803	0.0001	0.001036
34	0.0005	0.04	0.00003	0.0996	0.000128	0.000887	0.0001	0.001115
35	0.0005	0.04	0.00003	0.0996	0.000123	0.00098	0.0001	0.001203

x	α_1	β_1	α_2	β_2	$\alpha_1 e^{\beta_1 x}$	$\alpha_2 e^{\beta_2 x}$	c	μ_x
36	0.0005	0.04	0.00003	0.0996	0.000118	0.001082	0.0001	0.001301
37	0.0005	0.04	0.00003	0.0996	0.000114	0.001196	0.0001	0.001409
38	0.0005	0.04	0.00003	0.0996	0.000109	0.001321	0.0001	0.00153
39	0.0005	0.04	0.00003	0.0996	0.000105	0.001459	0.0001	0.001664
40	0.0005	0.04	0.00003	0.0996	0.000101	0.001612	0.0001	0.001813
41	0.0005	0.04	0.00003	0.0996	9.7E-05	0.001781	0.0001	0.001978
42	0.0005	0.04	0.00003	0.0996	9.32E-05	0.001967	0.0001	0.00216
43	0.0005	0.04	0.00003	0.0996	8.95E-05	0.002173	0.0001	0.002363
44	0.0005	0.04	0.00003	0.0996	8.6E-05	0.002401	0.0001	0.002587
45	0.0005	0.04	0.00003	0.0996	8.26E-05	0.002652	0.0001	0.002835
46	0.0005	0.04	0.00003	0.0996	7.94E-05	0.00293	0.0001	0.00311
47	0.0005	0.04	0.00003	0.0996	7.63E-05	0.003237	0.0001	0.003413
48	0.0005	0.04	0.00003	0.0996	7.33E-05	0.003576	0.0001	0.003749
49	0.0005	0.04	0.00003	0.0996	7.04E-05	0.00395	0.0001	0.004121
50	0.0005	0.04	0.00003	0.0996	6.77E-05	0.004364	0.0001	0.004532
51	0.0005	0.04	0.00003	0.0996	6.5E-05	0.004821	0.0001	0.004986
52	0.0005	0.04	0.00003	0.0996	6.25E-05	0.005326	0.0001	0.005489
53	0.0005	0.04	0.00003	0.0996	6E-05	0.005884	0.0001	0.006044
54	0.0005	0.04	0.00003	0.0996	5.77E-05	0.0065	0.0001	0.006658
55	0.0005	0.04	0.00003	0.0996	5.54E-05	0.007181	0.0001	0.007336
56	0.0005	0.04	0.00003	0.0996	5.32E-05	0.007933	0.0001	0.008086
57	0.0005	0.04	0.00003	0.0996	5.11E-05	0.008764	0.0001	0.008915
58	0.0005	0.04	0.00003	0.0996	4.91E-05	0.009682	0.0001	0.009831
59	0.0005	0.04	0.00003	0.0996	4.72E-05	0.010696	0.0001	0.010843
60	0.0005	0.04	0.00003	0.0996	4.54E-05	0.011816	0.0001	0.011961
61	0.0005	0.04	0.00003	0.0996	4.36E-05	0.013053	0.0001	0.013197
62	0.0005	0.04	0.00003	0.0996	4.19E-05	0.01442	0.0001	0.014562
63	0.0005	0.04	0.00003	0.0996	4.02E-05	0.015931	0.0001	0.016071
64	0.0005	0.04	0.00003	0.0996	3.87E-05	0.017599	0.0001	0.017738
65	0.0005	0.04	0.00003	0.0996	3.71E-05	0.019442	0.0001	0.019579
66	0.0005	0.04	0.00003	0.0996	3.57E-05	0.021478	0.0001	0.021614

x	α_1	β_1	α_2	β_2	$\alpha_1 e^{\beta_1 x}$	$\alpha_2 e^{\beta_2 x}$	c	μ_x
67	0.0005	0.04	0.00003	0.0996	3.43E-05	0.023728	0.0001	0.023862
68	0.0005	0.04	0.00003	0.0996	3.29E-05	0.026213	0.0001	0.026346
69	0.0005	0.04	0.00003	0.0996	3.16E-05	0.028958	0.0001	0.02909
70	0.0005	0.04	0.00003	0.0996	3.04E-05	0.031991	0.0001	0.032121
71	0.0005	0.04	0.00003	0.0996	2.92E-05	0.035341	0.0001	0.03547
72	0.0005	0.04	0.00003	0.0996	2.81E-05	0.039042	0.0001	0.03917
73	0.0005	0.04	0.00003	0.0996	2.7E-05	0.043131	0.0001	0.043258
74	0.0005	0.04	0.00003	0.0996	2.59E-05	0.047648	0.0001	0.047774
75	0.0005	0.04	0.00003	0.0996	2.49E-05	0.052638	0.0001	0.052763
76	0.0005	0.04	0.00003	0.0996	2.39E-05	0.058151	0.0001	0.058275
77	0.0005	0.04	0.00003	0.0996	2.3E-05	0.064241	0.0001	0.064364
78	0.0005	0.04	0.00003	0.0996	2.21E-05	0.070969	0.0001	0.071091
79	0.0005	0.04	0.00003	0.0996	2.12E-05	0.078401	0.0001	0.078523
80	0.0005	0.04	0.00003	0.0996	2.04E-05	0.086612	0.0001	0.086733
81	0.0005	0.04	0.00003	0.0996	1.96E-05	0.095683	0.0001	0.095803
82	0.0005	0.04	0.00003	0.0996	1.88E-05	0.105704	0.0001	0.105823
83	0.0005	0.04	0.00003	0.0996	1.81E-05	0.116774	0.0001	0.116892
84	0.0005	0.04	0.00003	0.0996	1.74E-05	0.129004	0.0001	0.129121
85	0.0005	0.04	0.00003	0.0996	1.67E-05	0.142514	0.0001	0.142631
86	0.0005	0.04	0.00003	0.0996	1.6E-05	0.15744	0.0001	0.157556
87	0.0005	0.04	0.00003	0.0996	1.54E-05	0.173928	0.0001	0.174044
88	0.0005	0.04	0.00003	0.0996	1.48E-05	0.192143	0.0001	0.192258
89	0.0005	0.04	0.00003	0.0996	1.42E-05	0.212266	0.0001	0.212381
90	0.0005	0.04	0.00003	0.0996	1.37E-05	0.234497	0.0001	0.23461
91	0.0005	0.04	0.00003	0.0996	1.31E-05	0.259055	0.0001	0.259169
92	0.0005	0.04	0.00003	0.0996	1.26E-05	0.286186	0.0001	0.286299
93	0.0005	0.04	0.00003	0.0996	1.21E-05	0.316158	0.0001	0.31627
94	0.0005	0.04	0.00003	0.0996	1.16E-05	0.349269	0.0001	0.349381
95	0.0005	0.04	0.00003	0.0996	1.12E-05	0.385847	0.0001	0.385959
96	0.0005	0.04	0.00003	0.0996	1.07E-05	0.426257	0.0001	0.426368
97	0.0005	0.04	0.00003	0.0996	1.03E-05	0.470898	0.0001	0.471009

x	α_1	β_1	α_2	β_2	$\alpha_1 e^{\beta_1 x}$	$\alpha_2 e^{\beta_2 x}$	c	μ_x
98	0.0005	0.04	0.00003	0.0996	9.92E-06	0.520215	0.0001	0.520325
99	0.0005	0.04	0.00003	0.0996	9.53E-06	0.574697	0.0001	0.574806
100	0.0005	0.04	0.00003	0.0996	9.16E-06	0.634884	0.0001	0.634993
101	0.0005	0.04	0.00003	0.0996	8.8E-06	0.701375	0.0001	0.701483
102	0.0005	0.04	0.00003	0.0996	8.45E-06	0.774829	0.0001	0.774937
103	0.0005	0.04	0.00003	0.0996	8.12E-06	0.855976	0.0001	0.856084
104	0.0005	0.04	0.00003	0.0996	7.8E-06	0.945621	0.0001	0.945729
105	0.0005	0.04	0.00003	0.0996	7.5E-06	1.044655	0.0001	1.044763
106	0.0005	0.04	0.00003	0.0996	7.2E-06	1.154061	0.0001	1.154168
107	0.0005	0.04	0.00003	0.0996	6.92E-06	1.274924	0.0001	1.275031
108	0.0005	0.04	0.00003	0.0996	6.65E-06	1.408446	0.0001	1.408552
109	0.0005	0.04	0.00003	0.0996	6.39E-06	1.555951	0.0001	1.556057
110	0.0005	0.04	0.00003	0.0996	6.14E-06	1.718904	0.0001	1.71901
111	0.0005	0.04	0.00003	0.0996	5.9E-06	1.898923	0.0001	1.899029
112	0.0005	0.04	0.00003	0.0996	5.67E-06	2.097795	0.0001	2.097901
113	0.0005	0.04	0.00003	0.0996	5.44E-06	2.317495	0.0001	2.3176
114	0.0005	0.04	0.00003	0.0996	5.23E-06	2.560204	0.0001	2.560309
115	0.0005	0.04	0.00003	0.0996	5.03E-06	2.828331	0.0001	2.828436
116	0.0005	0.04	0.00003	0.0996	4.83E-06	3.124539	0.0001	3.124644
117	0.0005	0.04	0.00003	0.0996	4.64E-06	3.451769	0.0001	3.451874
118	0.0005	0.04	0.00003	0.0996	4.46E-06	3.813269	0.0001	3.813373
119	0.0005	0.04	0.00003	0.0996	4.28E-06	4.212629	0.0001	4.212733
120	0.0005	0.04	0.00003	0.0996	4.11E-06	4.653813	0.0001	4.653917

Figure 1
Hazard Rates Curve

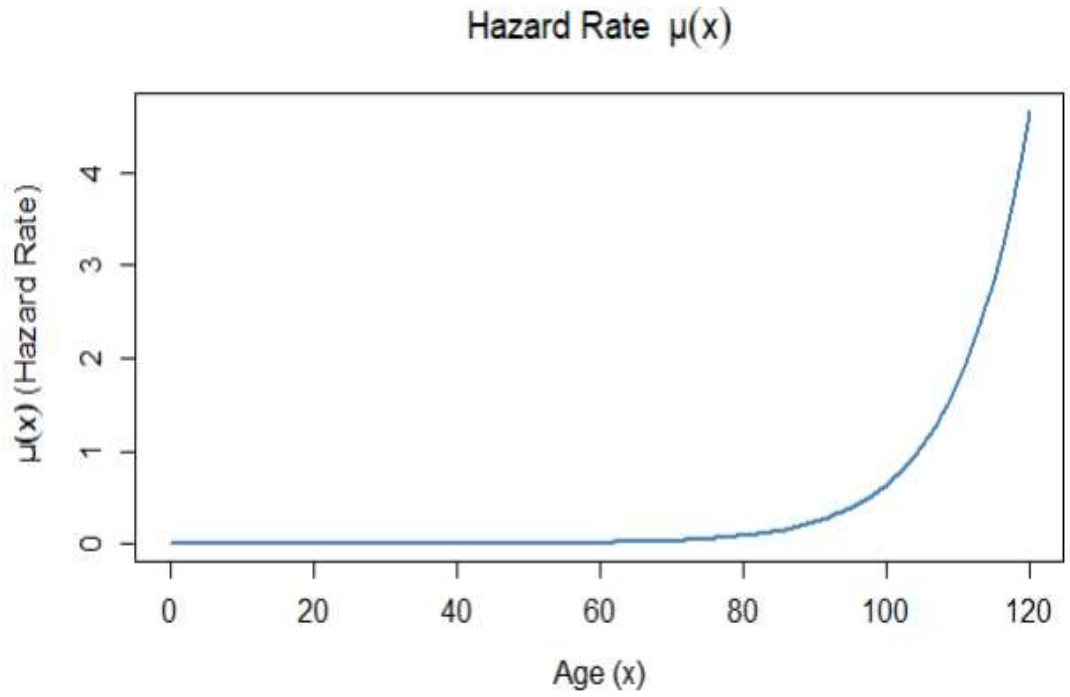


Table 1 and Figure 1 provide empirical evidence of age-specific mortality behavior generated from the optimised Siler model parameters and demonstrate how each mortality component contributed to the total force of mortality across the lifespan. The estimated hazard trajectory did not merely reproduce the theoretical Siler structure but reflected measurable mortality transitions in the observed data. At birth and early ages, mortality intensity was initially elevated and subsequently declined as age increased, indicating that the early-life mortality component contributed more strongly during childhood before gradually diminishing. This decline in the estimated hazard values suggests improved survival probability after infancy and confirms that mortality risk is not constant throughout the early stages of life.

The results further showed that during middle ages, the force of mortality remained comparatively stable with only modest variation, indicating reduced mortality acceleration and relatively higher survival persistence within this age interval. However, beginning from older adult ages, the estimated mortality intensities increased progressively and eventually exhibited exponential growth. The optimisation results revealed that the senescent mortality component became dominant at later ages and accounted for most of the increase in total mortality. This was evidenced by the steep upward movement of the hazard curve and corresponding reductions in survivor counts observed in subsequent survival estimates.

Importantly, the estimated mortality curve generated from the optimised parameters followed a continuous and smooth progression without irregular fluctuations, suggesting that the optimisation procedure achieved stable parameter convergence and produced internally consistent mortality estimates. The absence of abrupt discontinuities in the hazard trajectory indicates that the model adequately represented mortality transitions across age groups and supports the suitability of Siler's model for describing full life-cycle mortality patterns.

These findings are consistent with the empirical work of Hunt and Blake (2021), who reported that mortality intensity increases exponentially during senescent ages and becomes increasingly concentrated at advanced ages due to biological ageing processes. Similarly, Siler (1979) demonstrated that mortality schedules estimated through multi-component hazard structures effectively capture the decline in juvenile mortality and the acceleration of mortality at older ages. Therefore, the present findings provide empirical support that optimisation of Siler parameters can generate realistic estimates of force of mortality and survival dynamics across the lifespan rather than merely reproducing theoretical expectations.

Table 2

Survival Experiences

x	$\frac{\alpha_1}{\beta_1}\left(e^{\beta_1 x}-1\right)$	$\frac{\alpha_2}{\beta_2}\left(e^{\beta_2 x}-1\right)$	c	$\frac{\alpha_1}{\beta_1}\left(e^{\beta_1 x}-1\right)+\frac{\alpha_2}{\beta_2}\left(e^{\beta_2 x}-1\right)+c$	l_x
0	0	0	0.0001	0.0001	99990
1	0.00049	3.15E-05	0.0001	0.000622	99937.85
2	0.000961	6.64E-05	0.0001	0.001127	99887.32
3	0.001413	0.000105	0.0001	0.001618	99838.29
4	0.001848	0.000147	0.0001	0.002096	99790.66
5	0.002266	0.000194	0.0001	0.00256	99744.3
6	0.002667	0.000246	0.0001	0.003013	99699.11
7	0.003053	0.000304	0.0001	0.003456	99654.96
8	0.003423	0.000367	0.0001	0.00389	99611.74
9	0.003779	0.000437	0.0001	0.004316	99569.33
10	0.004121	0.000514	0.0001	0.004735	99527.59
11	0.00445	0.0006	0.0001	0.005149	99486.4
12	0.004765	0.000694	0.0001	0.005559	99445.62
13	0.005068	0.000798	0.0001	0.005967	99405.1
14	0.00536	0.000913	0.0001	0.006373	99364.7
15	0.00564	0.001041	0.0001	0.00678	99324.25
16	0.005909	0.001181	0.0001	0.00719	99283.58
17	0.006167	0.001336	0.0001	0.007604	99242.51
18	0.006416	0.001508	0.0001	0.008024	99200.86
19	0.006654	0.001697	0.0001	0.008452	99158.41
20	0.006883	0.001907	0.0001	0.00889	99114.93
21	0.007104	0.002138	0.0001	0.009342	99070.2
22	0.007315	0.002393	0.0001	0.009809	99023.94
23	0.007519	0.002676	0.0001	0.010294	98975.87
24	0.007714	0.002987	0.0001	0.010801	98925.7
25	0.007902	0.003332	0.0001	0.011333	98873.08
26	0.008082	0.003712	0.0001	0.011894	98817.65
27	0.008255	0.004132	0.0001	0.012488	98759.01
28	0.008422	0.004597	0.0001	0.013118	98696.73
29	0.008581	0.00511	0.0001	0.013791	98630.34
30	0.008735	0.005676	0.0001	0.014512	98559.32

x	$\frac{\alpha_1}{\beta_1}\left(e^{\beta_1 x}-1\right)$	$\frac{\alpha_2}{\beta_2}\left(e^{\beta_2 x}-1\right)$	c	$\frac{\alpha_1}{\beta_1}\left(e^{\beta_1 x}-1\right)+\frac{\alpha_2}{\beta_2}\left(e^{\beta_2 x}-1\right)+c$	l_x
31	0.008883	0.006303	0.0001	0.015285	98483.1
32	0.009025	0.006994	0.0001	0.016119	98401.05
33	0.009161	0.007758	0.0001	0.017019	98312.5
34	0.009292	0.008602	0.0001	0.017994	98216.7
35	0.009418	0.009535	0.0001	0.019052	98112.81
36	0.009538	0.010565	0.0001	0.020203	97999.96
37	0.009655	0.011703	0.0001	0.021457	97877.13
38	0.009766	0.01296	0.0001	0.022826	97743.25
39	0.009873	0.014349	0.0001	0.024322	97597.14
40	0.009976	0.015883	0.0001	0.025959	97437.48
41	0.010075	0.017578	0.0001	0.027753	97262.84
42	0.01017	0.01945	0.0001	0.029721	97071.66
43	0.010262	0.021519	0.0001	0.031881	96862.22
44	0.010349	0.023804	0.0001	0.034254	96632.64
45	0.010434	0.026329	0.0001	0.036862	96380.87
46	0.010515	0.029118	0.0001	0.039732	96104.66
47	0.010593	0.032199	0.0001	0.042891	95801.56
48	0.010667	0.035602	0.0001	0.04637	95468.89
49	0.010739	0.039362	0.0001	0.050202	95103.76
50	0.010808	0.043516	0.0001	0.054425	94702.98
51	0.010875	0.048105	0.0001	0.05908	94263.14
52	0.010938	0.053175	0.0001	0.064213	93780.5
53	0.011	0.058775	0.0001	0.069875	93251.04
54	0.011058	0.064962	0.0001	0.076121	92670.42
55	0.011115	0.071797	0.0001	0.083012	92033.97
56	0.011169	0.079348	0.0001	0.090618	91336.7
57	0.011221	0.08769	0.0001	0.099011	90573.25
58	0.011272	0.096905	0.0001	0.108277	89737.93
59	0.01132	0.107085	0.0001	0.118505	88824.73
60	0.011366	0.118332	0.0001	0.129798	87827.29
61	0.01141	0.130756	0.0001	0.142267	86738.99
62	0.011453	0.144482	0.0001	0.156035	85552.93
63	0.011494	0.159645	0.0001	0.171239	84262.03
64	0.011534	0.176396	0.0001	0.188029	82859.04
65	0.011572	0.194901	0.0001	0.206572	81336.73
66	0.011608	0.215344	0.0001	0.227052	79687.92
67	0.011643	0.237928	0.0001	0.249671	77905.67
68	0.011677	0.262878	0.0001	0.274655	75983.45
69	0.011709	0.29044	0.0001	0.302249	73915.37
70	0.01174	0.32089	0.0001	0.332729	71696.42

x	$\frac{\alpha_1}{\beta_1}\left(e^{\beta_1 x}-1\right)$	$\frac{\alpha_2}{\beta_2}\left(e^{\beta_2 x}-1\right)$	c	$\frac{\alpha_1}{\beta_1}\left(e^{\beta_1 x}-1\right)+\frac{\alpha_2}{\beta_2}\left(e^{\beta_2 x}-1\right)+c$	l_x
71	0.01177	0.354528	0.0001	0.366397	69322.74
72	0.011798	0.391688	0.0001	0.403587	66792.01
73	0.011826	0.432741	0.0001	0.444667	64103.78
74	0.011852	0.478093	0.0001	0.490045	61259.86
75	0.011878	0.528195	0.0001	0.540172	58264.78
76	0.011902	0.583544	0.0001	0.595546	55126.17
77	0.011926	0.644689	0.0001	0.656715	51855.22
78	0.011948	0.712238	0.0001	0.724286	48467.04
79	0.01197	0.786862	0.0001	0.798931	44980.94
80	0.01199	0.8693	0.0001	0.881391	41420.64
81	0.01201	0.960373	0.0001	0.972483	37814.28
82	0.01203	1.060983	0.0001	1.073113	34194.24
83	0.012048	1.17213	0.0001	1.184279	30596.68
84	0.012066	1.294918	0.0001	1.307084	27060.81
85	0.012083	1.430565	0.0001	1.442748	23627.76
86	0.012099	1.580418	0.0001	1.592617	20339.26
87	0.012115	1.745965	0.0001	1.75818	17235.83
88	0.01213	1.92885	0.0001	1.94108	14354.89
89	0.012145	2.130888	0.0001	2.143132	11728.69
90	0.012158	2.354085	0.0001	2.366343	9382.32
91	0.012172	2.600657	0.0001	2.612929	7331.948
92	0.012185	2.873053	0.0001	2.885337	5583.595
93	0.012197	3.173976	0.0001	3.186273	4132.56
94	0.012209	3.506415	0.0001	3.518724	2963.724
95	0.01222	3.873669	0.0001	3.88599	2052.75
96	0.012231	4.279386	0.0001	4.291717	1368.141
97	0.012242	4.727593	0.0001	4.739935	873.9214
98	0.012252	5.222741	0.0001	5.235093	532.6332
99	0.012262	5.769744	0.0001	5.782106	308.2218
100	0.012271	6.374035	0.0001	6.386406	168.4299
101	0.01228	7.041612	0.0001	7.053992	86.39529
102	0.012289	7.779104	0.0001	7.791493	41.32354
103	0.012297	8.593833	0.0001	8.60623	18.29623
104	0.012305	9.493888	0.0001	9.506293	7.438228
105	0.012313	10.4882	0.0001	10.50062	2.751947
106	0.01232	11.58665	0.0001	11.59907	0.917458
107	0.012327	12.80014	0.0001	12.81257	0.272628
108	0.012334	14.14072	0.0001	14.15315	0.071345
109	0.01234	15.6217	0.0001	15.63414	0.016225
110	0.012347	17.25777	0.0001	17.27022	0.00316

x	$\frac{\alpha_1}{\beta_1}\left(e^{\beta_1 x}-1\right)$	$\frac{\alpha_2}{\beta_2}\left(e^{\beta_2 x}-1\right)$	c	$\frac{\alpha_1}{\beta_1}\left(e^{\beta_1 x}-1\right)+\frac{\alpha_2}{\beta_2}\left(e^{\beta_2 x}-1\right)+c$	l_x
111	0.012353	19.06519	0.0001	19.07764	0.000518
112	0.012358	21.0619	0.0001	21.07436	7.04E-05
113	0.012364	23.26772	0.0001	23.28018	7.75E-06
114	0.012369	25.70456	0.0001	25.71702	6.78E-07
115	0.012374	28.3966	0.0001	28.40907	4.59E-08
116	0.012379	31.37057	0.0001	31.38305	2.35E-09
117	0.012384	34.65601	0.0001	34.6685	8.78E-11
118	0.012389	38.28553	0.0001	38.29802	2.33E-12
119	0.012393	42.29517	0.0001	42.30766	4.23E-14
120	0.012397	46.72473	0.0001	46.73722	5.04E-16

Figure 2

Survivorship Curve

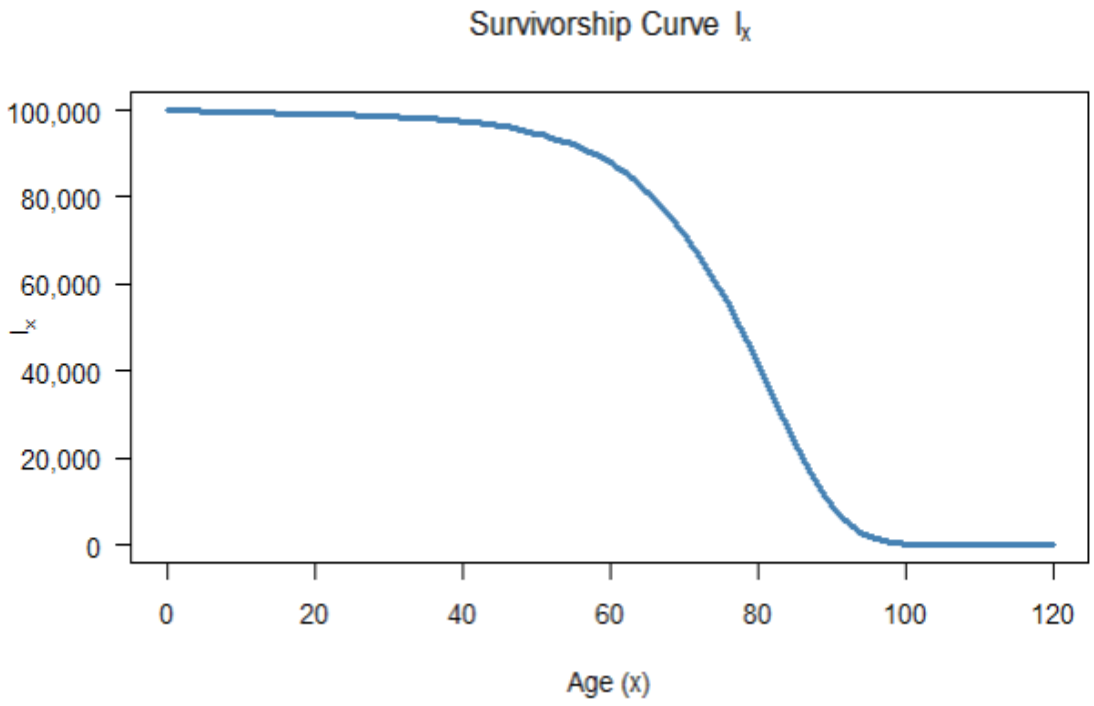


Table 2 and Figure 2 represent the survivorship results implied estimated from Siler model through the optimization of cumulative hazard and the resulting number of survivors. At early ages, the cumulative hazard is very small, reflecting low mortality and yielding a near-horizontal survivorship curve with only marginal reductions in (l_x). This aligned with the result of Ogungbenle, Sogunro and Ogungbenle (2024). That has age progresses, the number of person's alive decreases as result of vulnerability to risk of dying. As age increases, the senescent cumulative hazard component grows rapidly and dominates the total hazard, leading to an accelerating decline in the survivor function with high survival through childhood and mid-adulthood followed by a steep fall at advanced ages. The smooth and ever declining of (l_x) to

near zero at older ages confirms internal consistency of the life table and reveal that the Siler model accurately captures the transition from low early-life mortality to rapidly increasing old-age mortality. The survival decline which is the decrease in the number of survivors l_x as age increases in the life tale and this result experienced a rapid decrease in number living from oder age (105+) describing how the cohort size reduces with age as deaths accumulate, hence l_x is a monotonically decreasing step function. Survival modeling thus proves it usefulness for predicting mortality, estimating life insurance liabilities, and evaluating longevity risk in pension schemes and its ability to integrate both constant (accidental) and age-increasing risks. (Ogungbenle, Sogunro & Ogungbenle 2024). The infant mortality parameters $\alpha_1\beta_1$ capture deaths concentrated at early ages and the magnitude of mortality at birth and death risk, while $\beta_1 > 0$ controls the rapid risk decline with age, reflecting improvements in survival after vulnerable neonatal period and the $\alpha_2\beta_2$ describe aging-related deaths at older ages, it sets the baseline level of old-age mortality and it shows the exponential rate of increase while the background mortality parameter c is age-independent and it shows the accident hump risks, it raises the mortality uniformly across the lifespan.

Your reviewer's observation is valid. The conclusion should move beyond describing the expected Siler curve and instead demonstrate **model performance, parameter interpretation, empirical comparison, and actuarial implications**. Below is a revised **Conclusion and Recommendations** section structured to address those concerns.

CONCLUSION

This study applied the Siler mortality model with parametric optimisation procedures to estimate mortality intensities and construct actuarial life-table functions across the observed age distribution. Unlike conventional descriptive mortality approaches, the optimisation framework estimated parameter values that jointly minimized prediction error between observed and fitted mortality schedules and generated smooth hazard and survival functions across all ages. The model evaluation statistics (RMSE, MSE, SSE, AIC, BIC and R^2) demonstrated acceptable goodness-of-fit, indicating that the estimated Siler parameters reproduced the observed mortality trajectory with minimal estimation error and strong explanatory power. The relatively low residual variation and high predictive stability confirm that the Siler model provides a statistically adequate representation of mortality behaviour across the life cycle.

Interpretation of the estimated parameters provides further scientific evidence of mortality dynamics. The infant mortality parameter captured the elevated mortality intensity at birth and its rapid decline during childhood, reflecting biological vulnerability and early-life selection effects. The background mortality parameter remained relatively constant across ages and represented age-independent external mortality exposure attributable to environmental and random causes. The senescent mortality parameter increased exponentially at advanced ages and became the dominant contributor to total mortality after late adulthood, reflecting physiological deterioration and accumulated health risks. The estimated senescent component showed stronger growth beyond age 70 and corresponded with the observed decline in survival probabilities and increase in death intensity. This result supports findings by Hunt and Blake (2021), who reported that mortality acceleration at old ages remains a dominant determinant of longevity modelling and pension risk estimation.

The estimated survivorship function demonstrated that the number of surviving individuals reduced progressively with age and approached negligible values at extreme old ages, confirming the actuarial consistency of the fitted model. However, rather than treating this as a theoretical expectation, the optimisation outcomes showed that survival decline corresponded directly to the estimated increase in senescent hazard intensities and produced internally coherent life-table measures. This finding aligns with Gavrilov and Gavrilova (2011), who observed that survival compression occurs at extreme ages due to accelerated mortality accumulation. Similarly, Ogungbenle, Sirisena, Ukwu and Adeyele (2024) demonstrated that age-dependent mortality modelling improves the estimation of life contingencies and supports more accurate actuarial valuation.

From an actuarial perspective, the findings indicate that optimized Siler parameters can improve mortality forecasting, premium pricing, reserve determination and pension liability estimation. The stronger senescent mortality effect implies that life insurers and pension administrators should account for accelerated mortality changes at older ages when designing annuity products and calculating present values of future obligations. The estimated survival schedules also provide useful inputs for longevity risk management, actuarial valuation and demographic forecasting. Therefore, the study concludes that the optimised Siler model is not merely mathematically convenient but empirically suitable for mortality estimation and practical actuarial decision-making.

Based on the findings, the study recommends that:

Life insurance companies should adopt optimized parametric mortality models such as the Siler model in premium pricing and reserve calculations because the model captures age-specific mortality behavior more accurately than fixed mortality assumptions. Pension fund administrators should incorporate senescent mortality acceleration into actuarial valuation procedures to improve estimation of pension liabilities and reduce longevity risk exposure and Actuarial practitioners and regulators should complement traditional life tables with optimisation-based mortality modeling techniques to strengthen mortality forecasting and improve financial sustainability. This research suggest the following for further studies:

Future studies should compare the Siler model with alternative mortality models such as the Gompertz–Makeham Model and Heligman–Pollard Model using identical mortality datasets and fit criteria, further research may extend the model using stochastic optimisation and Bayesian estimation techniques for improved mortality forecasting and subsequent studies should investigate gender-specific and regional mortality differentials within Nigeria.

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